The Golden & Silver Ratio: An Overview

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Abstract: throughout the history, the metallic ratios had been used widely in the fields of architecture, mathematics, painting, designing etc. The two metallic ratios, golden & silver ratios have several applications. These metallic spirals can be observed almost everywhere in nature. Even in human body and in body structures of some animals, these spirals can be observed. From the ancient civilizations to modern age the ratios are being used several times by the architects, designers, painters and even sometimes by photographers.

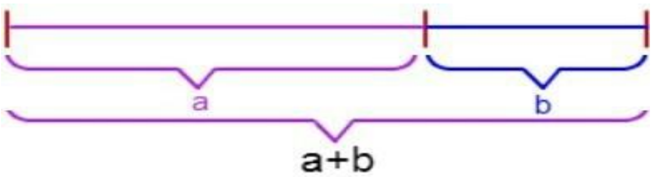
KEYWORDS: Silver ratio; Golden ratio, Fibonacci sequence; Spiral

I INTRODUCTION

Have you ever thought about the spirals observed in many places of nature? Or why some architectures & paintings appear so pleasant to our eyes? Actually, there is an important role played by two ratios (golden & silver ratio) behind the pleasant appearances. So, let us know about them. The aim of the article is to provide some knowledge about golden and silver ratio among readers.

II THE GOLDEN RATIO

Golden ratio, also known as the Golden mean, Golden section, Divine proportion etc. is the special irrational number $\frac{1+\sqrt{5}}{2} \approx 1.618$, which is denoted by the Greek letter φ.

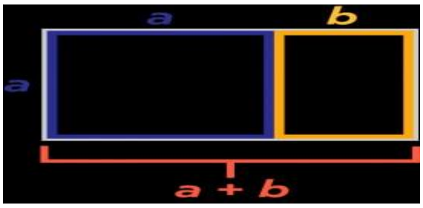


Let us consider the above straight line, which is divided in two different lengths a & b. The total length of the line is $a + b$. Now the two different quantities a and b are said to be in golden ratio if the ratio of the larger part and the smaller part is equal to the ratio of the total length of the line and both the ratios are equal to 1.618 i.e. if $\frac{a}{b} = \frac{a+b}{a} = 1.618$.

IIA THE GOLDEN RATIO & GEOMETRY:

Golden ratio can be found in geometric shapes like rectangle, triangle, pentagon. It makes frequent and unexpected appearance in geometry.

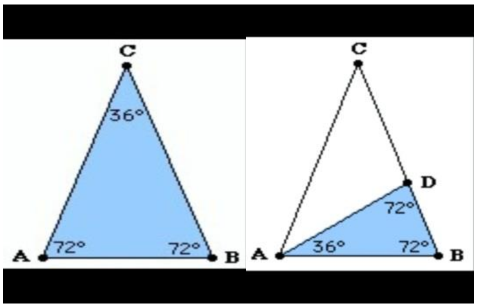
Golden rectangle: Golden rectangles are those rectangles whose sides are in Golden ratio. Let us consider the following rectangle.



The length is $a + b$ and the width is b. If $(a + b)/a = a/b = 1.618$, then the above rectangle is said to be a golden rectangle.

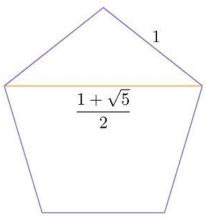
The interesting fact is that a golden rectangle can be divided in a square and an another rectangle and the divided rectangle is also a golden rectangle i.e. if we consider ABCD as a golden rectangle, which is divided into a square (length of each side $= a$) and a rectangle(length$= a$, width $= b$) , then the divided rectangle (length$= a$, width$= b$) is also a golden rectangle. Each golden rectangle can be divided into a square and another golden rectangle. Thus, a golden rectangle can contain many golden rectangles.

Golden triangle: Golden triangle is an isosceles triangle whose each equal side and base is in golden ratio. Each base angel is 72degree and the other angel is 36 degree. If one of base angel is bisected, the bisector divides the opposite side of the angel in golden ratio and it divides the triangle in two triangles, one of which is also a golden triangle. Thus, a golden triangle also contains many golden triangles.



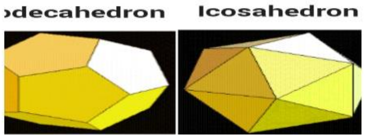
Here, ABC is a golden triangle. So, $AC: BC = BD: CD = 1.618$.

Golden pentagram: Golden ratio can be found in regular pentagons.

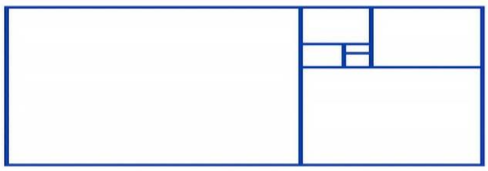


Regular pentagons are pentagons whose length of each side is equal. The ratio of first diagonal and side is φ.

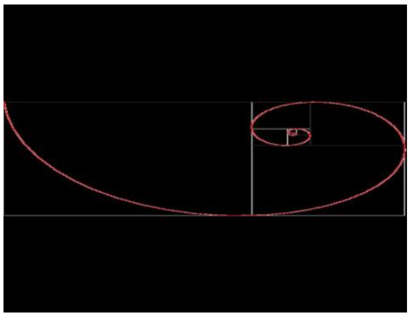
Golden ratio also appears in 3D geometric shapes. e.g. Dodecahedron (it has 12 faces and each face is a regular pentagon) and Icosahedrons (it has 20 faces; each face is a golden triangle.)



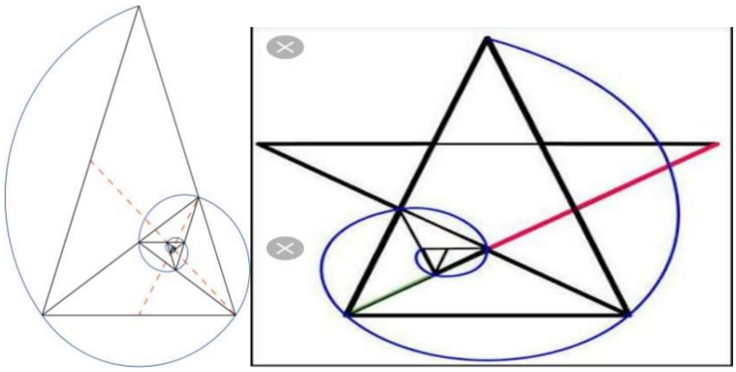
The Golden Spiral: We have read how a golden rectangle can contain many golden rectangles. By continuing the process of dividing the following image can be obtained.



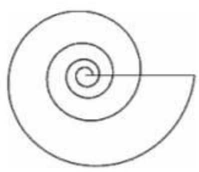
Now, if a line can be spiralled through the golden rectangle, the following spiral will appear which is known as the Golden spiral.



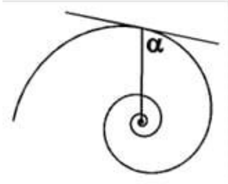
Similarly, the spiral can be found in golden triangle, golden pentagram.



Actually, the golden spiral is a special case of the logarithmic spiral.



Logarithmic spiral



The Golden spiral

The feature of the logarithmic spiral is, for each 90 degree turn of the spiral the distance from the centre of the spiral is multiplied with a fixed number. If for any case, the fixed number is φ, then it forms the golden spiral.

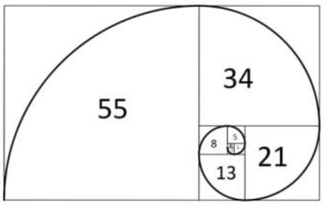
The angel made by the line from the centre of the logarithmic spiral to its tangent is always constant. So, logarithmic spiral is also called an *equiangular spiral*. In case of golden spiral, the angel is *72.96degree*, which is known as the *Golden angel*.

Fibonacci sequence and the Golden ratio: The Fibonacci sequence was invented by Italian Mathematician Leonardo Pisano Bigollo (1180-1250).The sequence is deeply related with the golden ratio. The first two terms of the sequence are 0&1. Then each term is just sum of it's previous two terms i.e. the sequence is,

0,1,1,2,3,5,8,13,21,34, ….

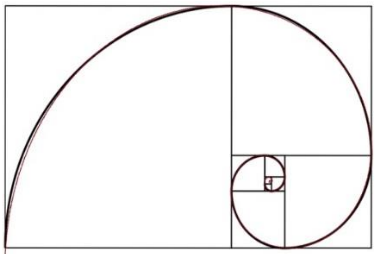
The interesting fact is that if we continue calculating the ratio of each term and its previous term, the ratio comes closer and closer to the golden ratio as the sequence grows.

Now, a question appears, "Is Fibonacci spiral is Golden spiral?" The answer is 'no' because Fibonacci spiral is drawn through the squares, whose length of the sides is taken from the consecutive terms Fibonacci sequence.



The Fibonacci spiral

But the Golden spiral is drawn through the golden rectangle.



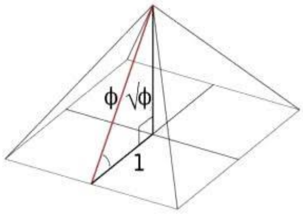
The Golden Spiral

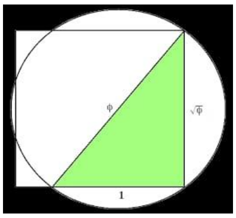
IIB HISTORY OF DISCOVERY OF THE GOLDEN RATIO & ITS APPLICATION

The golden ratio was applied again and again by the famous sculptors, painters, architects throughout the history for its unique and pleasing beauty. So, it is really hard to when and how the ratio was first discovered. It is assumed that the ancient Egyptians and Greeks first applied the concept of golden ratio. So, let us know about the discovery and application of it by going through the following data,

The Great Pyramid of Giza (around 2560 B.C.):

The Great Pyramid of Giza (Located in Giza, Egypt), one of the Seven Wonders of the World, built around 2560 B.C. was constructed using the Golden ratio.

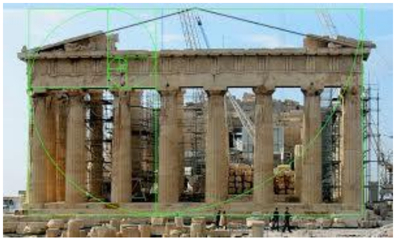




Kepler's Triangle

Each triangle of the pyramid satisfies the law of the Kepler's Triangle.
The Great Pyramid was constructed using φ, but those are not perfect golden triangles. To be a perfect golden triangle the slant height should be 186.367m [which differs just (186.369 – 186.367) =0.002m from the original slant height] as (186.367/115.182) = 1.61803

The Parthenon (447B.C. - 432B.C.):



The Parthenon, located in Athens, Greece is an ideal example of application of golden rectangle. It was designed by the great sculptor Phidias.

The Parthenon was constructed using the golden rectangle. The length of the beam at the bottom of the roof and each column of it are in Golden ratio. Even the design of the supporting beam and the floor plan of the Parthenon were designed with golden rectangles.
Great philosopher Plato (428B.C.-347B.C.) in his book 'Timaeus' (written in 360B.C.) about natural science and cosmology stated that,
"For whenever in any three numbers, whether cube or square, there is a mean, which is to the last term what the first term is to it: and again, when the mean is to the first term as the last term is to the mean- then the mean becoming first and last, and the first and last both becoming means, they will all of them of necessarily come to be the same, and having become the same with one another will be all one."

It is said that Plato described about the Golden ratio by this quote, though it is debatable.
Great mathematician Euclid (325B.C. – 265B.C.) describes in his *"Elements"(*published in 300B.C.) how a line can be divided in extreme and mean ratio i.e. if we consider a line AB, then a point C can be found such that AB: AC = AC : CB. But the term 'golden' could not be found in the book.

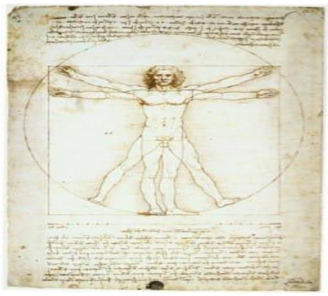
Invention of the Fibonacci Sequence(around1200A.D.):

The Fibonacci sequence was invented around **1200A.D**. by Italian mathematician Leonardo Pisano Bigollo (also known as Leonardo Fibonacci). He wrote about the Fibonacci sequence in his book "Liber Abaci", where a problem about the birth of rabbit colony was solved by using the sequence. But the relationship between the sequence and φ was still unknown.

Leonardo da Vinci & The Divine Proportion:

The book "De Diviana Proportione" (The Divine Proportion), was written by mathematician Luca Pacioli and was published in 1509. The book contains many beautiful illustrations of 3D geometric solids and templates for script letters in calligraphy. The book also explains the application mathematics in art, which inspired the Italian polymath Leonardo da Vinci and he applied golden ratio widely in his paintings. About three of them are discussed in the following.

The Vitruvian Man:



It is a late 15th century drawing by Leonardo da Vinci which represents a perfect proportion of human body. The radius of the circle and the side of the square which is equal to the height of the man are in golden ratio.

The Last Supper:

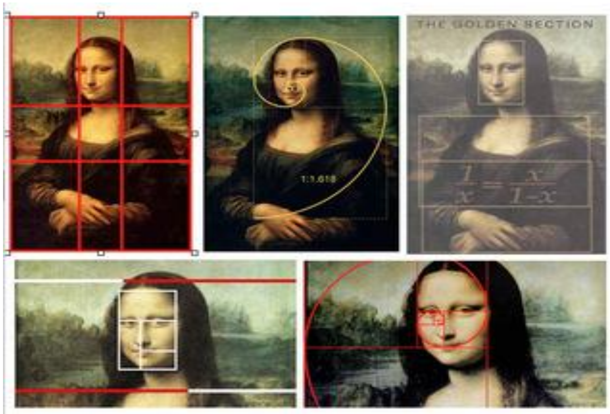
The last supper is a painting of Jesus having supper with his disciples, drawn between 1494 & 1498 by Leonardo Da Vinci. Starting from the centre of the table, every small shield, even the top of windows displays the Golden ratio very clearly.

The centre of the table Small shields

It is also said that the position of each disciple around the table to Jesus was fixed in φ.

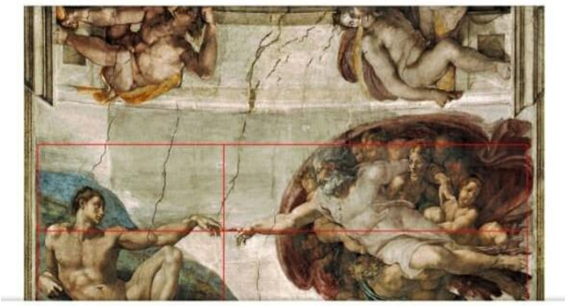
The Mona Lisa:

"Monalisa" is the one of the greatest creations of Leonardo da Vinci, probably painted sometime between 1503 to 1506. But it is also said that it was incomplete till 1516.



Golden ratio was applied throughout the painting. Many golden rectangles can be found in the painting e.g. if a rectangle is drawn around her face it will appear as a golden rectangle. If it is divided by a line across her eyes another golden rectangle can be found. In this way many golden rectangles can be found on the rest of the painting as shown in the picture.

The Creation of Adam:

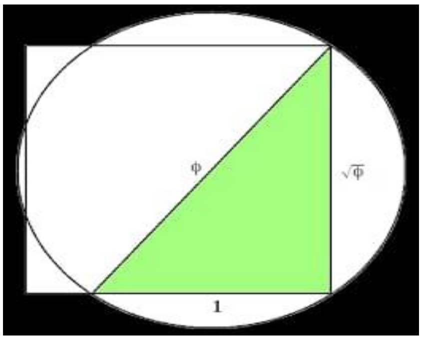


Famous Italian painter Michelangelo (1475-1564) applied Golden ratio in his painting "The Creation of Adam", drawn in 1511-1512A.D. Golden rectangles can be clearly shown in that part of the picture where Adam is reaching to God.

The School of Athens:

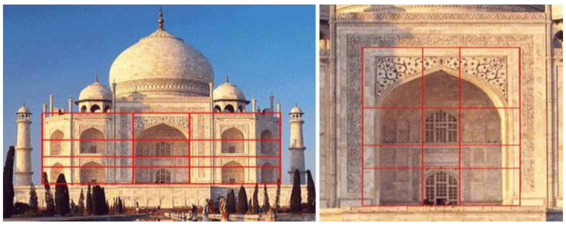
Wonderful application of Golden rectangles can be seen in the painting “The School of Athens” by the great Italian painter & architect Raffaello Sanzio da Urbino, known as Raphael (1483-1520A.D.). It was drawn between 1509 to 1511A.D.

The Kepler’s Triangle:



Kepler’s Triangle is a right-angle triangle whose lengths of the sides are in progression $1{:}\sqrt{\varphi}{:}\,\varphi$, was invented by German Astronomer Johannes Kepler (1571-1630). According to him,
“Geometry has two great treasures: one is the theorem of Pythagoras; and the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel.”

The Tajmahal (1653A.D.):



The Taj Mahal, excellent representation of architecture in Agra, India, one of the Seven Wonders of the World was designed using the Golden ratio. It was completely built in **1653** by Mughal Empire Sahjahan. The main building of the Taj Mahal was completely designed using golden rectangle. The ratio of the building and the width of the central arc is φ.

Goldener schnitt:

In 1815, famous German mathematician Martin Ohm (1792-1872) published a book named “Die Reine Elementar-Mathematik” (The Pure Elementary Mathematics) about the golden ratio and it was the first book where the term ‘Goldener Schnitt’ (Golden Section) was used. So, it is believed that the term ‘Golden Ratio’ was coined by Martin Ohm. This book can be said a turning point in the world of mathematics. The English term was first used in 1875 by James Sulley in 9th edition of his article of aesthetics ‘Encyclopaedia Britannica’.

The Golden Stairs(1876-1880A.D.):

The Golden Stairs drawn by well-known artist Edward Burne Jones is one of the ideal examples of golden ratio in art. It was drawn in 1876 and was exhibited in 1880.The golden staircase in this picture represents a golden spiral. Even the dresses of the women on the staircase were also drawn using golden proportion.

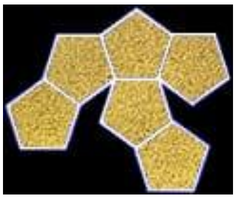
Use of the Greek alphabet φ:

In the beginning of 20th century American mathematician Mark Barr first used the Greek alphabet φ[‘phi’] according to the name of great sculptor Phidias and mathematician Fibonacci. It was then used in the ‘The Curves of Life’ by Theodore Andrea Cook in 1914.

Penrose Tiling-the amazing invention(1970A.D.):

Tiling is a method of covering a plane with a Particular Geometric shape in such a way that no gap should occur between two tiles (geometric shapes) and the tiles should not overlap to each other. There are two types of tiling, periodic and non-periodic.

We all know that a plane can be covered easily with squares, rectangles, triangles, hexagon etc. but tiling is not possible with pentagons, as shown below-

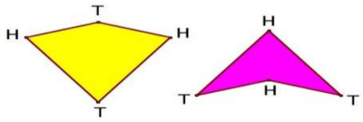


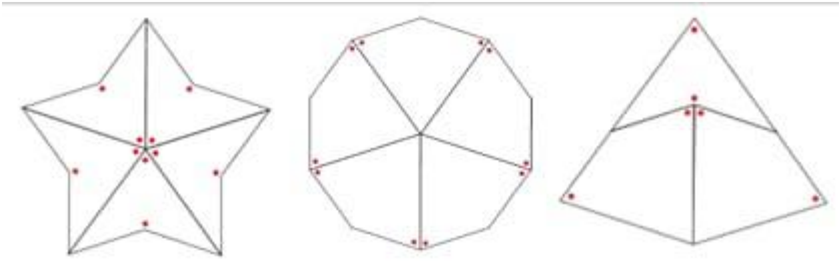
Gap occurs while covering a plane with pentagon.

In 1970, well known British Mathematician Roger Penrose made it possible by using the Golden ratio. He invented the 5-fold symmetry. The shapes used for this tiling are known as Penrose Tiles and this method of tiling using Penrose Tiles is known as Penrose Tiling. This is a non-periodic tiling.

There are many types of Penrose tiling. But two of the most general are Kite&Dart and Rhombic Penrose Tiles.

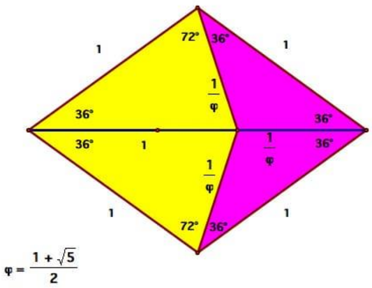
Kite&Dart: The set of Penrose Tiles, shown above are known as Kite and Dart. Here, the yellow coloured tile is Kite and the pink coloured is Dart. These shapes can be used to cover a plane in the following ways called ‘Star’, ‘Sun’ & ‘ace’.

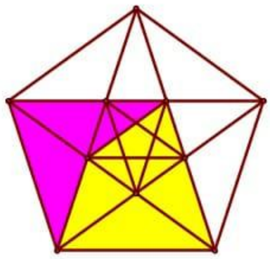




Star Sun Ace

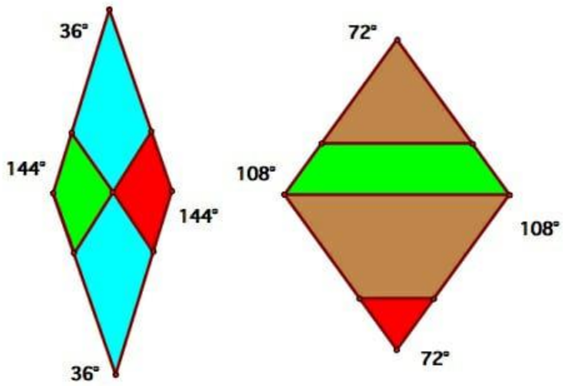
In case of Star and Sun only Dart and Kite are used respectively. In case of ace, the Kite and Dart are used together. Now the interesting fact is that, the ratio of the areas in any Penrose Tiling covered by the Kite and the Dart are always φ. The sides of the Kite and Dart are also in φ. Even in any Penrose Tiling, the number of kites and the number of Darts are always in φ.





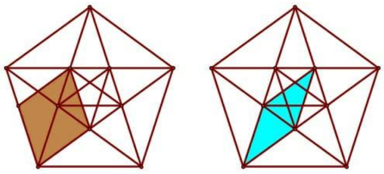
Look at the above pentagon. It was discussed before that the sides and the diagonals of a regular pentagon are in φ. In every regular pentagon a Kite and Dart can be found in the above way.

Rhombic Penrose Tiling:

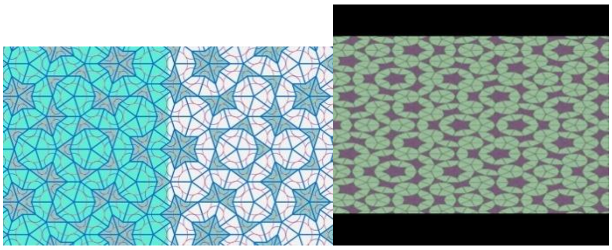


Thin rhombus Fat rhombus

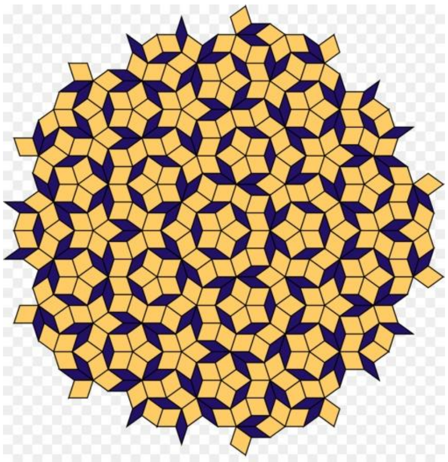
This is another set of Penrose Tiling. It is combination of two types of rhombus- fat rhombus and thin rhombus. The ratio of the length of each side and the diagonal is φ, for both of them. Like Kite and Dart, number of fat rhombus and the number of thin rhombus used in this tiling is always φ. The areas covered by the tiles are also in φ. Like Kite and Dart these two rhombuses can be found in a regular pentagon, as shown below-



Here are three beautiful images of Penrose tiling.



Penrose Tiling (Kite and Dart)



Rhombic Penrose Tiling

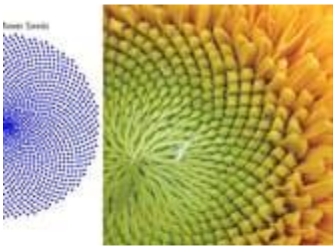
Toronto's CN Tower (1976A.D.):

CN Tower, the largest free-standing statue of the world, located in Toronto, Canada was designed using the Golden ratio. It was built within 40 months and was opened to the citizens on 26th June, 1976. I t was built for communication. It is the centre of telecommunication in Canada. From the observation deck 62-75 miles distance can be observed. It is the landmark of Canada [CN stands for Canadian Nation]. The height of the tower and the observation deck from land is 1815ft (=553.33m) and 342m respectively. Now, 342/553.33 = 0.618, the reciprocal of φ i.e. the total height of the tower and the height of the observation deck are in Golden ratio.

IIC GOLDEN RATIO IN NATURE

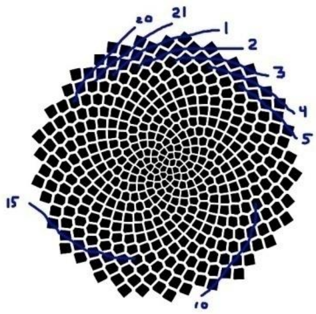
It is already discussed before how the Fibonacci spiral and the Golden spiral are deeply related to each other. It is really a great surprise to us that the Fibonacci sequence and the golden spiral can be seen almost everywhere in nature. Starting from the growth structure of leaves, flower petals, seeds of many plants to the sea wave, the golden spiral can be observed even in physical structure of some animals and even in human body. As it is like God's signature in his own creations, the Golden ratio is also known as the "God's Ratio". Here are some examples where the Golden spirals and Fibonacci numbers can be observed in nature.

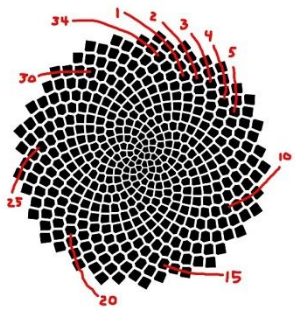
Sunflower seeds:



Spiral growth of Sunflower seeds

Golden ratio can be observed in spiral growth of sunflower seeds, towards clockwise and anticlockwise. In each sunflower number of spiral arms whose direction of spiral growth is clockwise and the number of spiral arms whose spiral growth is anticlockwise are always in φ, as they are always two consecutive terms of the Fibonacci sequence.





Spiral growth in clockwise direction   Spiral growth in anticlockwise direction

In the above pictures, direction of spiral growth of 21 arms is clockwise and 34 arms are anticlockwise. Now 21 and 34 are consecutive Fibonacci numbers. If we stare at those sunflower seeds the Golden spiral growth of seeds can be observed easily. The arrangement of the seeds according to the golden angle is also responsible for this beautiful pattern.

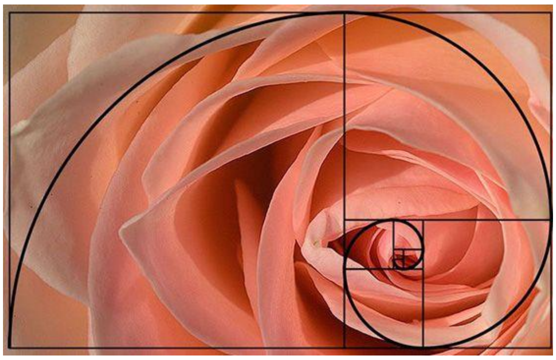
Golden Ratio in other plants and flowers:

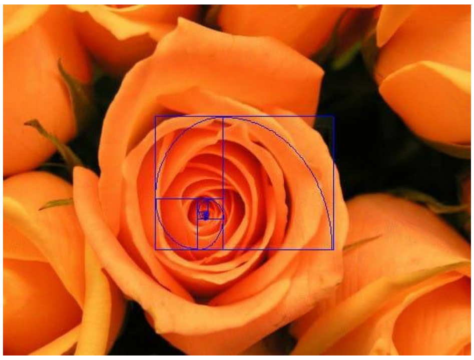
There are species of flowers where Fibonacci sequence is followed in their number of petals, as shown below.

Name of flowers	Number of petals
Lily, Iris	3
Buttercup, wild rose, larkspur	5
delphiniums	8
Corn marigold cineraria	13
Aster, chicory	21

pyrethrum	34
Michelmas daisies	55

The pleasant Golden spiral can also be observed in the pattern of some flower petals. Let's see the spiral growth of the queen of flower, Rose.





Spiral growth of the Rose petals

The growth of the Rose petals follows the Fibonacci sequence. Growth of each set of petals is dependent on the growth of its previous set of petals. So, the petals towards outside are larger than the petals towards the inside i.e. the growth of the petals are arranged according to the Fibonacci sequence in descending order from the outside of the flower towards inside of it. Also, the gap between the set of petals follows the Fibonacci sequence i.e. the petals towards inside of Rose are closer to each other than the petals towards outside. So, the spiral nature of Rose represents the Golden spiral. Golden spiral can be observed in spiral nature of growth in the following plants in the similar manner.
Spiral growth of the leaves of Polyphylla (also known as Spiral Aloe).

The Spiral Aloe

Growth of the Fern fiddleheads:

Spiral nature of Fern

If a tangent can be drawn in any point of the spiral, the angle made by the line from the centre of the spiral to the tangent will be the Golden angel. So, it represents the golden spiral.

Pine Cones:

Pine Cones

Like the Sunflower seeds, the seeds of Pine Cones follow the Fibonacci numbers and spiral growth.

Spiral growth of Calla Lily:

Spiral growth of Calla Lily

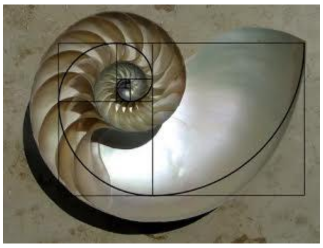
The flower Calla Lily contains only one petal, which grows exactly like the Golden spiral. The angle made by the line from the centre of the petal to the tangent (if drawn) at any point on the curve of the petal is always the Golden angel.

This spiral growth of these leaves and flower makes them beautiful to our eyes and for this every petal and every leaf from inside to outside of the spiral get equal sunlight.

Starfish:

The physical structure of Starfish displays a regular pentagon, each side and the diagonals are in golden ratio, as discussed before.

Similarly, the Golden Spiral can be observed in Ocean wave, Nautilus Shells and Conch Shells as shown below.


Spiral ocean wave Golden spiral in Nautilus shell Golden spiral in Conch shell

II D SPIRAL GALAXY

According to South African researches, Golden Ratio can also be found in Universe. Spiral Galaxy is one of the examples of them.

Look at the following picture. Each 90 degree turn of the spiral galaxy is multiplied by φ with previous turn, as the definition of the Golden spiral.

Spiral Galaxy

II E GOLDEN RATIO IN HUMAN BODY:

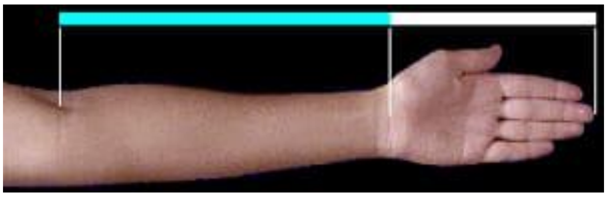
Human body contains many ratios from head to feet. Most of these are very close to the Golden Ratio.

First of all, see, the number of our hands (2), the number of fingers in each hand (5), number of legs (2), number of toes in each leg(5), number of sense of organs(5) are all Fibonacci numbers.

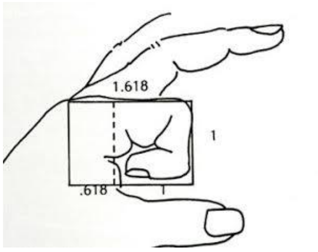
Now let us discuss about the distances between some body parts which are in φ.

a) From the head to fingertips with total height of our body are very close to the Golden ratio.
b) Distance from the head to navel with the distance from the head to fingertips;
c) Distance from head to shoulders with the distance from head to fingertips;
d) Distance from the head to chin with distance from the head to shoulders; all of these are very close to φ.

Golden ratio in hands:



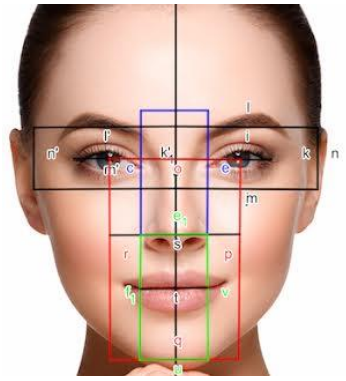
a) Ratio of the distance from elbow to longest fingertip and the length from the elbow to the base of wrist is very close to φ.
b) Length of forearm and the total length of our hand are in φ.
c) Golden ratio can also be found in structure of wrist and fingers as shown below.



Golden rectangle in finger

Each finger is divided into three parts and for the first finger, the lengths of the first two parts are in φ and for this cause they can form the golden rectangle, as shown in the above image.

φ in our face



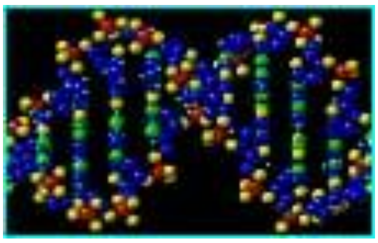
Golden Proportion in our face

The following ratios of our face are very close to φ,

a) Ratio of the length and width of the face.
b) Ratio of the distance from lips to middle of eyebrows and the length of nose.
c) Ratio of the distance between two black dots of our eyes and the distance between two ends of the eyebrows.

Human DNA

DNA, in which all physical features are stored, shows golden ratio in its structure. For each full cycle of its double helix spiral, the length and the width of DNA molecule is 34 angstroms and 21 angstroms respectively. Now, 21 7 34 are two consecutive Fibonacci numbers and 34/21 = 1.619, very close to φ.



III THE SILVER RATIO

The Silver ratio, also known as the Silver mean is the irrational number $1+\sqrt{2} = 2.4142135\ldots \approx 2.414$.

Let us consider the above straight line (total length is 2A+B) which is divided into three segments where each of the two larger segments is equal to A and length of the smaller segment is equal to B. Now, the two different numbers A and B are said to be in silver ratio if,

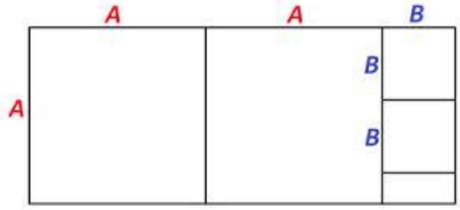
$$A/B = (2A+B)/A = 2.414$$

Thus, two different quantities are said to be in silver ratio if, the ratio of the lager & the smaller quantity is equal to the ratio of {2(larger quantity) + smaller quantity} & the larger quantity and both of them are equal to 2.414. The Silver ratio is denoted by ∂s.

III A THE SILVER RATIO AND GEOMETRY

The silver rectangle:

The concept is almost like the Golden rectangle. If the sides of a rectangles are in silver ratio, it is called the silver rectangle.

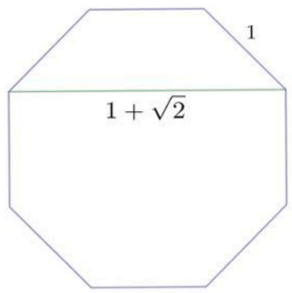


$$\frac{2A+B}{A} = \frac{A}{B} = \frac{B}{A-2B}$$

Silver rectangle

The interesting fact is that, if two largest possible squares of same areas are removed from a silver rectangle, a small rectangle will be left, which is also a silver rectangle and this silver rectangle can also be divided into two squares of equal length of their sides and another silver rectangle, as shown in the above image. Since, each silver rectangle can be divided in two squares of same area & another silver rectangle, many silver rectangles can be found in one silver rectangle, if the process of the division is continued.

Silver Octagon:

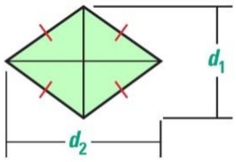


Silver octagon

As we saw, that φ is associated with regular pentagon, the silver ratio is associated with regular octagon (an octagon, whose each side length is equal).The ratio of second diagonal and side is 2.414.

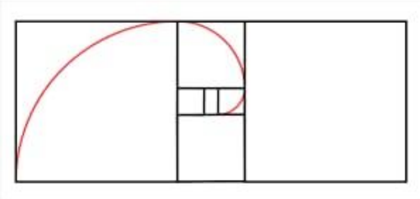
Silver Rhombus:

Silver rhombus is the rhombus, whose diagonals are in silver ratio.



If we consider the above rhombus as a silver rhombus, then $d_2/d_1 = 2.414$. The acute and obtuse angle of a silver rectangle is 70 degree31' and 109degree 28' respectively.

The Silver spiral:



The Silver spiral

If a line is spiralled through one of the squares and the remaining rectangle, of a silver rectangle, a spiral can be obtained which is known as the Silver spiral, as shown above.

The silver spiral is also a special case of the Logarithmic spiral, like the Golden Spiral. In case of Silver spiral, for each 90 degree turn, distance from the centre of the spiral is multiplied with 2.414.

Silver ratio from Pell numbers:

Let us again remember the Fibonacci sequence,

$$0, 1, 1, 2, 3, 5, 8, 13, 21 \ldots$$

Clearly, if the n^{th} term of the sequence is P_n then,

$$\mathbf{P_n = P_{n-1} * 1 + P_{n-2}}, \text{ for n=3, 4, 5...}$$

Now, the Pell sequence is,

$$0, 1, 2, 5, 12, 29, 70, 169 \ldots$$

$\Rightarrow \mathbf{P_n = P_{n-1} * 2 + P_{n-2}}$, for n= 3, 4, 5…

Now, if we continue calculating each term with its previous term, the ratio comes closer and closer to 2.414.

III B APPLICATIONS IN HISTORY

Like the Golden ratio, silver ratio has many applications throughout the history. But the fact is that, it was mainly used in Japan, Asia. According to Japanese, the silver ratio is more pleasant than the Golden ratio. So, it is also known as ***"Yamato-hi",*** means ***"Japanese Ratio"***.

Silver ratio in architecture:

i) The Shitenno-Ji Temple: It is one of the oldest Buddhist temples of Japan, located in Osaka. It was constructed using the silver ratio in 593 AD by Prince Shotoku.

Inner precinct of Shitenno-Ji Temple

The 5-storey pagoda of the inner precinct is designed using the silver ratio. The ratio of the length of the first roof and the last roof of it is silver ratio. The silver rectangles can be found throughout the structures of the buildings of the temple. Clearly, the gap between those pillars of the building's forms silver rectangles.

ii) The Horyu-Ji Temple: The Horyu-Ji Temple, built in 607AD in Asuka period by Prince Shotoku, located in Nara of Japan. It is the oldest wooden building of the world. It is a Buddhist temple. The Horyu-Ji complex is consisted with 48 listed buildings, western precinct, and Eastern precinct.

The western precinct of the Horyu-ji temple

The western precinct consists of main hall (the left building of the above image), five storey pagoda (the right building of the above image) and central gate (at the centre of the picture). The first roof of the five storey pagoda (roof of the ground floor) and its last roof (roof of the 4th floor) are in silver ratio. The ground floor and the 1st floor of the main hall displays silver rectangles in their structures. The widths of those two rectangles are also in silver ratio.

The Eastern precinct of the Horyu-ji temple

Many silver rectangles were used in construction of the Eastern precinct. Even the base of the building was designed using the Silver rectangles.

In 1993, the temple was enlisted in the UNESCO Wold Heritage Site.

iii) The Kinkakuji Temple: The Kinkakuji Temple, located in Kyoto, Japan, was built in 1397. It is also known as the Golden Pavilion. It is an ideal example of application of silver rectangles in architecture. The top two floors are completely covered with gold leaf.

The Kinkakuji Temple

The temple was completely designed with silver rectangles. Structure of each floor, even the golden boundaries displays clear silver rectangles.

iv) The Ginkakuji Temple: The Ginkakuji temple is also located in Kyoto, Japan. It was Built in 1482 CE. It is known as the silver pavilion, also called the cousin of the Kinkakuji temple.

The Ginkakuji Temple

Like the Kinkakuji Temple, the Ginkakuji Temple is also designed with silver rectangles.

Silver ratio in painting:

The following paintings were drawn using the silver ratio.

i) Beauty Looking Back: The "Beauty Looking Back" was drawn by famous Japanese painter Hishikawa Moronobu (1618-1694). It is a picture of a beautiful woman in red Kimono, who is looking back.

Beauty Looking Back

The picture of the woman is within a rectangle of length 63cm and width 31.2cm. Now, 63/31.2 = 2.019, very close to the Silver ratio.

ii) Landscapes of Autumn & Winter: It is two famous paintings, both designed using the silver ratio by famous Japanese painter Sesshu (1420-1509).

Landscape of Autumn

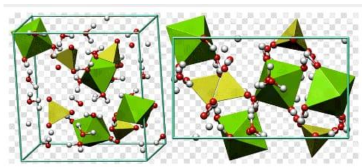
Landscape of winter

III C SILVER RATIO IN NATURE

Like the Golden ratio, the Silver ratio can also be found in Nature. Here are some examples of it.

i) Zinc and Magnesium: Zinc and Magnesium are two of the most required minerals for our Immune system, muscle strength and metabolism. Both of their structure displays the 3D geometric figure Cub octahedron.

Zinc



Magnesium sulphate

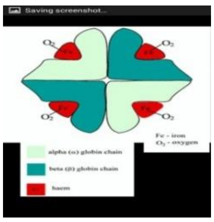
ii) Shell of scallop:

Shell of scallop

The shell of scallop is an ideal example of silver spiral in nature. If a tangent is drawn at any point of silver spiral, the angle created by the line joining the centre of the silver spiral to the point, with the tangent is 61degree which can also be obtained in those shells by the same way.

III D SILVER RATIO IN HUMAN BODY

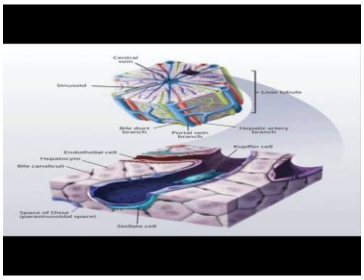
i) Haemoglobins of blood:



Structure of Haemoglobin

The structure of Haemoglobin of our blood displays a regular octagon.

ii) Cell of the Liver:



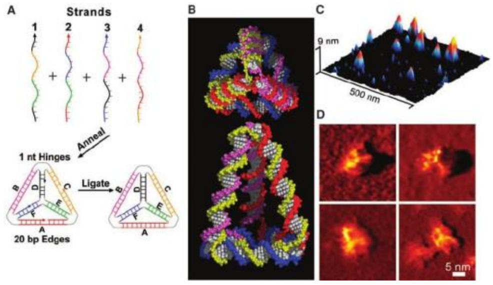
Cell structure of liver

The cells structure of our liver display rhombic dodecahedrons.

IV SOME OTHER APPLICATIONS OF THE GOLDEN AND THE SILVER RATIO

Here are some other applications of golden and silver ratio.

Tetrahedron model of DNA:



Tetrahedron model of DNA

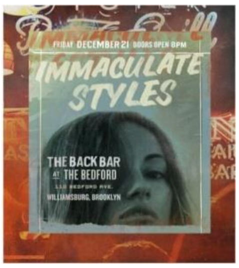
The tetrahedron model of DNA was invented by physicists of UK. Each side of the tetrahedron is made of double helix structure of DNA. Now, silver ratio appears in tetrahedron. The tetrahedron model is used widely in the field of Medical Science.

Graphic designing:

Now a days, Graphic designing is a very popular course among young stars. The graphic designers often use golden ratio in their designs.

Typography hierarchy: The concept of the golden ratio is very useful in this field. Suppose, while making a poster you are confused about the correct font size to be used at the body. Now, if the font used by you is multiplied with 1.618, a standard font size can be obtained. Let a designer is making a poster or cover of a book with 10pts font size. Now, $10 * 1.618 = 16.18 \approx 16$, which should be applied for the best font size. Now, if someone uses 20pt font size for the heading, then it should be divided by 1.618, as the font size of heading should be larger than the body text. Also, in case of poster making, to determine best size of poster, the concept of the golden rectangle and golden spiral is applied. Here is a poster which is designed using the Golden ratio.

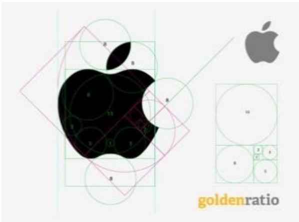
Beautiful midsummer poster by green in blue



Gritty poster for DJ Immaculate Styles by nevergohungry

According to the graphic designers, while applying the golden spiral in poster, the main thing which is to be shown to people must be put into the centre of the spiral, as it easily draws the attention of people.

Logo designing: Many famous companies used the golden ratio in their logo design. The graphic designers use φ by putting the design of logo within golden rectangles. Generally, they design the logos in such a way that golden spiral can be found in the logos by very careful observation. Here are some examples of logos which were designed using the golden ratio.



Logo of Apple Company

Logo of Grupo boticario

Logo of Pepsi company

Cloud logo

φ in photography:

φ is used widely in the field of photography. Golden rectangle is used to determine best size of a photo. Photographers often crop an image in such a way that the cropped image becomes a golden rectangle, which helps to attract people easily. Here are two examples of cropping photo by using golden ratio.

Floral designing:

Perfect proportion and scale are two of the most important principles in floral designing, which can be maintained easily by using the golden proportion and the silver ratio. In Western floral designing, golden ratio is used by the Fibonacci sequence. The Japanese floral designing is known as “Ikebana”, which is similarly, designed using the Pell sequence and silver ratio.

Architecture:

Now a days also many buildings are constructed using the concept of golden ratio, as it is aesthetically pleasant.

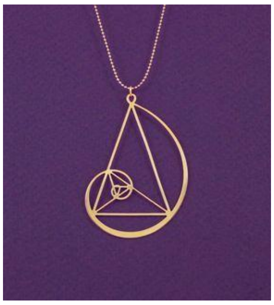
Use of the concept of Parthenon

Use of Golden rectangles

vi) Jewellery design:

Now a days many beautiful jewelleries are designed by using golden ratio and silver ratio.

Golden spiral locket



Golden triangle and golden spiral

pentagon and the golden spiral

Jewellery design using Silver rhombus

V CONCLUSION

The two metallic ratios are deeply related to almost every field of our life and especially with our natural world. But, according to some designer, painters and architects those metallic ratios are not so pleasant. Even many artists do not like applying the golden ratio in their painting. But if we see the role of these two ratios in our life, clearly it opens a new door of learning.

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